MA/MSCMT-09

June - Examination 2016

M.A./M.Sc. (Final) Mathematics Examination

Integral Transforms and Integral Equations

Paper - MA/MSCMT-09

Time : 3 Hours] **[Max. Marks :- 80**

Note: The question paper is divided into three sections A, B and C. Write answers as per given instructions.

Section - A $8 \times 2 = 16$

(Very Short Answer Questions)

Note: Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) State existence conditions of the Laplace transform.

(ii) State Initial Value Theorem for the Laplace transform.

(iii) What are the convergence conditions of a Fourier Series?

(iv) State Boundary Value Problem.

(v) Write definition of Mellin Transform.

(vi) Write relation between Hankel and Laplace transform.

(vii) Define linear and non linear integral equations.

(viii) Define Degenerate Kernel.

Section - B **$4 \times 8 = 32$**

(Short Answer Questions)

Note: Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Find the Laplace Transform of $f(t) = t^2 e^t \sin 4t$.

3) Find the complex Fourier Transform of $f(t) = e^{-a|t|}$, where $a > 0$ and t belongs to $(-\infty, \infty)$.

4) If $F(p)$ and $G(p)$ are the Mellin Transform of $f(x)$ and $g(x)$ respectively. Find the Mellin Transform of

$$x^{\lambda} \int_0^{\infty} u^{\mu} f\left(\frac{x}{u}\right) g(u)\, du,$$ where λ and μ are constants.

5) Find the Hankel transform of the function

$$f(x) = \begin{cases} x^n, & 0 < x < a \\ 0, & x > a \end{cases} \quad (n > -1)$$

taking $xJ_n(px)$ as the Kernel.

6) Show that the function $g(x) = \sin\left(\frac{\pi x}{2}\right)$ is a solution of the Fredholm integral equation

$$g(x) - \frac{\pi^2}{4} \int_0^1 K(x, t)\, g(t)\, dt = \frac{x}{2}.$$

7) Solve the homogeneous Fredholm integral equation.

$$\phi(x) = \lambda \int_0^1 e^{x+t} \phi(t)\, dt.$$

8) Solve : $\int_0^\infty f(x) \cos px\, dx = e^{-p}.$

9) Solve : $g(x) = x.2^x - \int_0^x 2^{x-t} g(t)\, dt,\ g_0(x) = x.2^x$ by using the method of successive approximations.

Section - C **2 × 16 = 32**

(Long Answer Questions)

Note: Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words. Use of non-programmable scientific calculator is allowed in this paper.

10) (i) State and prove convolution theorem for Laplace transform.

(ii) Find the inverse Laplace transform of $\log\left(1 + \frac{1}{p^2}\right)$.

11) Use parseval's identity to prove that

(i) $$\int_{-\infty}^{\infty} \frac{dt}{(a^2 + t^2)(b^2 + t^2)} = \frac{\pi}{2ab(a + b)}, \quad (a > 0, b > 0)$$

(ii) $$\int_{-\infty}^{\infty} \frac{\sin at}{(a^2 + t^2)}\, dt = \frac{\pi}{2}\left\{\frac{1 - e^{-a^2}}{a^2}\right\}$$

12) Find the eigenvalues and eigenfunctions of the homogeneous integral equation.

$$g(x) = \lambda \int_0^{\pi} [\cos^2 x \cos 2t + \cos 3x \cos^3 t]\, g(t)\, dt.$$

13) By Iterative method, solve $g(x) = 1 + \lambda \int_0^{\pi} \sin(x + t) g(t)\, dt.$
